

WORKFORCE CONFIGURATION OF A CANADIAN FORCES GEOMATICS DIVISION

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Abstract

This paper addresses some of the pertinent issues related to the workforce configuration of a C2 organization within the Canadian Forces. The mission of the latter is to produce Geomatics Information supporting US National Imaging and Mapping Agency's (NIMA) Foundation Based Operations (FBO).

Initially, the open queueing network representation of the Geomatics division (where each node or station is governed by a $GI/G/s$ queue) is examined and its complexity analyzed. The Geomatics network belongs to the class of queueing network with signals. An alternate network architecture is proposed and the intent of which is to provide a simplified network whereby the theory of product-form solutions can be employed to evaluate the workforce configuration. The equivalence of the L norm on the waiting times between the original and the revised network is demonstrated. A nonlinear integer programming model to minimize the L norm on the waiting times for the revised network is formulated. The solution procedure involves transforming the nonlinear problem into a linear problem using approximation techniques. Fictitious data are used to illustrate the methodology.

1. Introduction

This paper is based on the concept that a military intelligence organization has, as one of its functions, the requirement to process defined quantities of messages within stipulated periods. It is therefore of interest to test the ability of any given system to fulfil that requirement and to investigate means for improving the system efficiency. This paper outlines an approach to model the workforce configuration of the Canadian Forces (CF) Geomatics organization. The mission of the latter organization is to produce and deliver Global Geospatial Information & Services to the Canadian Forces that cannot be acquired elsewhere for reasons of uniqueness, urgency and security.

The discussion of this paper will focus on analyzing the processing of information for the Foundation Data Concept where its aim is to support the Canadian Forces' objectives in Global Preparedness, Theater Readiness and Mission Responsiveness. The Foundation Data Concept is a framework developed by the U.S. National Imaging and Mapping Agency (NIMA) in support of Joint Vision 2010 and it consists of geospatial information of sufficient data content and accuracy. The data can be grouped into 3 categories, namely the Foundation Data (FD), Mission Specific Data Sets (MSDS) and Qualified Data. The objective of the Foundation Data is to provide information useful for strategic operations and planning; whereas the Mission Specific Data Sets furnish useful information for tactical operations and planning. Qualified Data is an alternative source to FD and MSDS. In essence, the database holding FD will host information extracted from multiple sensor types collected from a variety of platforms obtained through multi-national sources. On the other hand, MSDS consists of higher resolution data and is also referred to as dense foundation data. The Foundation Data of interest in this paper are Controlled Image Base (CIB) with 5-meter resolution, Feature Foundation Data (FFD) and Digital Terrain Elevation Data (DTED) level 2 information with 30-meter resolution. The MSDS of interest here are the higher resolution CIB, DTED and densified FFD information products. The development of FFD and DTED information from raw source data requires a

lengthy time period to prepare, with FFD being the most expensive and difficult to produce. Depending on the area to be covered, the production of bare earth DTED data for a region typically takes months to complete. Therefore the development and sustainability of FD and MSDS database covering significant areas of the world will undoubtedly require a multi-national effort.

In general, there exist three main approaches to the analysis of an organization's workflow process. a). *Static (allocation) models*. These simply add up the total amount of jobs allotted to each resource, and estimate the performance from these totals. Such a model tends to be too simple as it ignores most of the dynamics, interactions and uncertainties of the system. On the other hand, it can be useful as a rough initial estimator of the system size and performance. b). *Aggregate dynamic models*. Such models account for some of the dynamics, interactions and uncertainties in the system in an aggregate way. Typically, they use analytical techniques from stochastic processes, queueing theory and queueing networks. However, the performance measures estimated are often only the steady state averages. Still, these models tend to give reasonable estimates of performance and relative to alternative approaches, are very efficient. c). *Detailed dynamic models*. These approaches include Petri Nets, Stochastic Petri Nets and Monte Carlo simulation. Petri nets are a graphical method of describing concurrent systems and have proven to be very useful for describing protocols in networks [Schneeweiss, 2001]. Discrete event simulation models, on the other hand, can mimic the operation of the system in as much detail as required and desired. Combined with visual animation, these models can be powerful tools for communicating the results of an analysis. However, apart from the problems of validating large and complex simulations, the complexity of the simulation models often results in limited insights into the factors determining behaviour. Rarely do complex simulation models suggest how the design or operation could be improved [Buzacott *et al.*, 1992].

This paper studies a CF Geomatics division, a typical Information Technology (IT) organization, by examining its network representation. The complexity of the network is discussed. An alternate network architecture is proposed. The equivalence of the L norm on the waiting times between the two networks is demonstrated. The intent of deriving an alternative network representation is such that the theory of product-form solutions [Gross and Harris, 1998], which renders the mathematics involved to be more tractable, can be applied to evaluate the optimal configuration. A nonlinear integer programming model to minimize the L norm on the waiting times is proposed. That is, the optimal allocation of servers to the nodes/stations such that the smallest upper bound on the waiting times is achieved, will be determined. Our solution procedure involves linearizing the nonlinear terms using a pre-determined probability related function in *LINGO6* software [Lindo System, 1999]. A sequence of linear integer programming problems is then solved iteratively. The LP relaxation to the integer problem is also discussed with an aim to gain useful information on the solution space and on post-optimality analysis.

2. Geomatics Organization - Network Representation

A typical Foundation Data project (job) involves the preparation of images for a region or a sub-region by extracting features and contour information from the input data. The production of images is affected by the following factors or variables, notably the image source (satellite image versus aerial photography), the image scale (surface covered by the image), the project area (for

example $100 \text{ km} \times 125 \text{ km}$), and the area density (quantity of information in an area) etc. The preparation routine for CIB, FFD and DTED consists of the setup, collection, extraction, edge matching and quality control/assurance operations. In brief, Figure 1 represents the pictorial architecture for the CF Geomatics division under investigation.

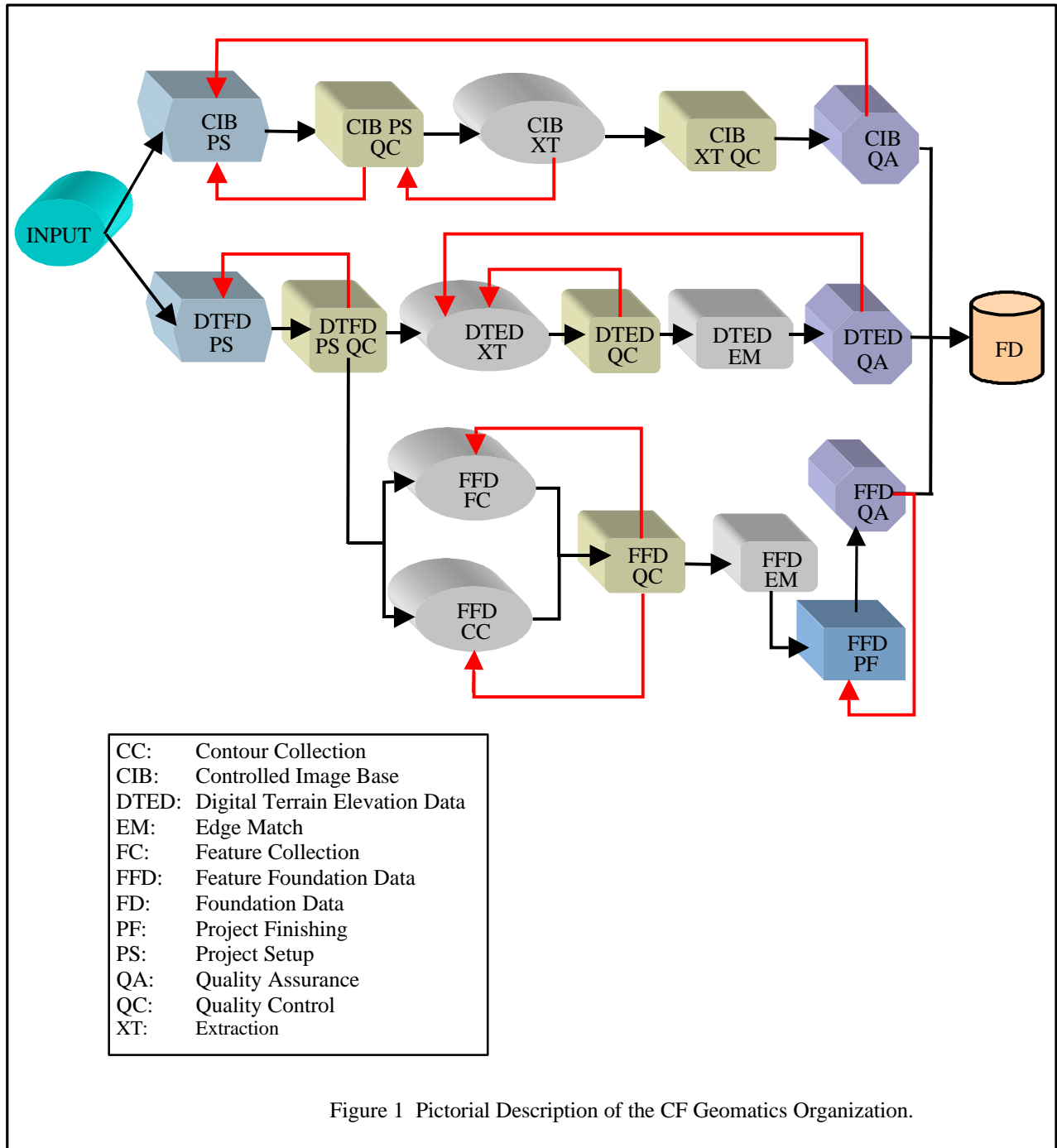
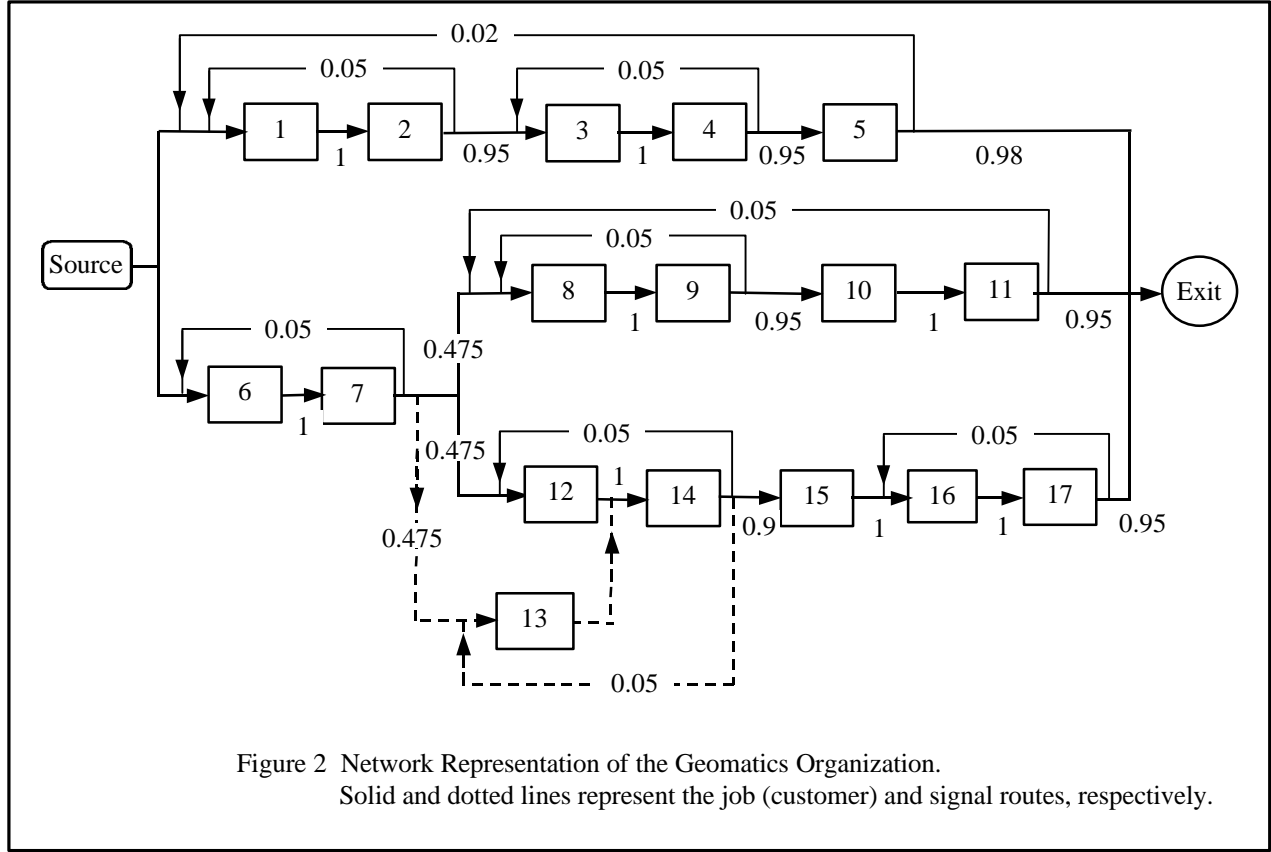


Figure 2 is the network representation of the division with the corresponding routing probabilities as indicated. The input data for the CF Geomatics system is summarized in Table 1.



The network (Figure 2) is interpreted as follows. The nodes or stations represent the subsections/operations of the organization. The network has N single-station queues and each station has s_i servers. It is assumed that there is an unlimited waiting space for the jobs at each station. At appropriate locations along the network, jobs/projects are inspected at the quality control/assurance stations and are subject to rework if required or warranted. Jobs finishing service at station i join the queue at station j with probability $p_{i,j}$ or leave the network altogether with probability r_i , independently of each other. Since a job leaving station i either joins some other station in the network, or leaves, one must have

$$\sum_{j=1}^{17} p_{i,j} + r_i = 1, \quad i = 1, 2, 3, \dots, 17 \quad (1)$$

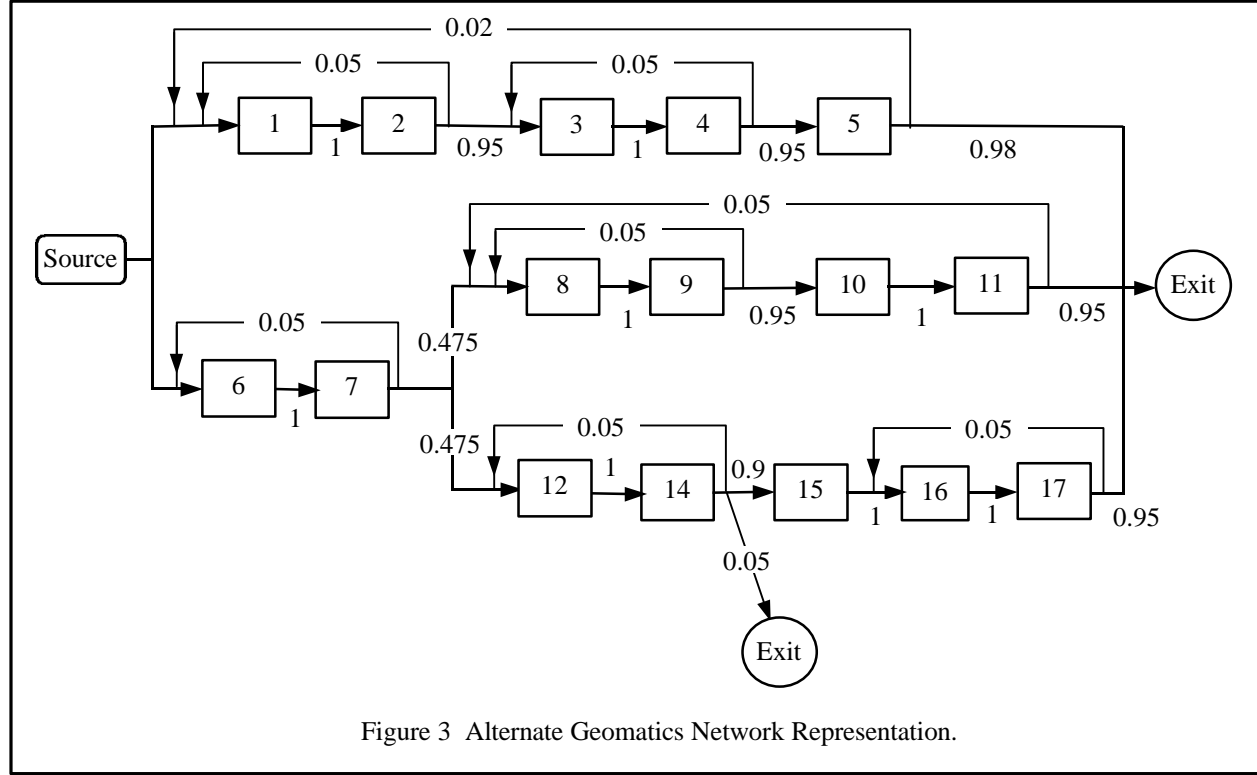
Consider the job processing sequence after the project setup node 7 (a procedure that involves creating the project database, calibrating the camera, importing & magnifying images and preparing for the image). A team of servers at node 12 will concentrate on preparing the feature collection for the image (that is preparing for the population, surface drainage, transportation and vegetation coverage). Simultaneously, another team of servers at node 13 will focus on contour collection, namely examine the boundary and elevation coverage.

Station (i)		External Arrival		Service	
		Rate (jobs/day)	Squared Coefficient of Variation	Rate (jobs/day)	Squared Coefficient of Variation
1	CIB	0.8	1.05	95	0.824
2		0	0	95	0.917
3		0	0	12	1.135
4		0	0	48	1.134
5		0	0	48	1.077
6	DTED / FFD	1.6	0.95	0.2	0.824
7		0	0	0.4	0.917
8	DTED	0	0	0.0126	1.135
9		0	0	0.05	1.134
10		0	0	0.2	1.077
11		0	0	0.065	0.804
12	FFD	0	0	0.04	1.044
13		0	0	0.04	1.044
14		0	0	0.09	0.916
15		0	0	0.3	1.145
16		0	0	0.16	0.973
17		0	0	0.21	0.806

Table 1 - Input Data for the Geomatics Organization

These teams use the same set of raw data input from the project setup node and the arrival rate of jobs at the feature collection and contour collection nodes (12 and 13) is identical. In addition, the job service times at these two nodes are approximately the same. Upon job completion, the two different jobs at nodes 12 and 13 merge as one job and are readied to be quality checked at node 14. The difficulty in analyzing the Geomatics network stems from the sudden increase and decrease in population size at nodes 7 and 14. That is at these nodes, jobs trigger simultaneous events and the network Figure 2 belongs to the class of networks of queues with signals [Chao *et al.*, 1999]. The latter class of queueing network involves, besides the regular jobs, also signals that carry commands and move around from node to node. When a signal arrives at a node, it causes a certain event to occur. This event may imply either the deletion of one or more jobs (for example at node 14 of Figure 2) or the addition of one or more jobs (at node 7 of Figure 2). The latter behaviour involving the sudden increase and decrease in population size is also symptomatic in many other information technology and intelligence processing organizations and industries.

The above qualitative description can be expressed mathematically by the routing probabilities at node 7. Since $p_{7,6} + p_{7,8} + p_{7,12} + p_{7,13}$ is greater than unity, it violates equality (1). Initially, our approach is to propose a reduction on the network architecture, given by Figure 3.



The network in Figure 3 is constructed in view of the following observation. It is noticed that the characteristics of the two branches connecting node 7 to node 14 via the arc through node 12 or via the arc through node 13 are identical. That is the transition probabilities $p_{7,12}$, $p_{7,13}$, the station service times and the feedback transition probabilities $p_{14,12}$, $p_{14,13}$ are the same for the two branches. The two networks only differ in that the branch from node 7 to node 14 via node 13 does not exist in Figure 3. On the other hand, Figure 3 has an arc (which is missing from Figure 2) connecting node 14 to an external node that exits the system, with r_{14} denoting the probability of jobs leaving the system from node 14. The transition probabilities at node 14 in Figure 3 satisfies $p_{14,15} + p_{14,12} + r_{14} = 1$, with the value r_{14} equal to $p_{14,13}$ in Figure 2.

It will be demonstrated that given the same configurations to the two systems, the L norms on the waiting times (that is the upper bound on the waiting times for all nodes in the network) are equivalent. The theory of product-form solutions can then be applied to analyze Figure 3.

Lemma 1. Given the same configurations to systems Figures 2 and 3, that is $(s_1, s_2, \dots, s_{17})$ with $s_{13} = s_{12}$ in Figure 2 and $(s_1, s_2, \dots, s_{12}, s_{14}, \dots, s_{17})$ in Figure 3, the L norms on the waiting times between two systems are equivalent.

Proof: Define $W_q^{\wedge} = L$ norm on $W_{q,i}$ in Figure 2, $1 \leq i \leq 17$
 $W_q^* = L$ norm on $W_{q,j}$ in Figure 3, $1 \leq j \leq 17, j \neq 13$

where $W_{q,i}$ is the steady state time a job spends at node i in the queue. The proof is trivial by virtue of the fact that the characteristics at nodes 12 and 13 are the same, that is $W_{q,12} = W_{q,13}$.

First, $W_q^{\wedge} \geq W_{q,i}, \quad 1 \leq i \leq 17, \quad \text{by definition}$
 $= W_{q,j}, \quad j=1,2,\dots,12,14,\dots,17$

Thus $W_q^{\wedge} \geq W_q^*$

On the other hand,

$W_q^* \geq W_{q,j}, \quad j=1,2,\dots,12,14,\dots,17$
 $= W_{q,i}, \quad 1 \leq i \leq 17,$

or $W_q^* \geq W_q^{\wedge}.$

Combining the two inequalities, $W_q^{\wedge} = W_q^*.$

3. Mathematical Model

Given the arrival process of jobs to nodes 1, 6 of the Geomatics division (Figure 3) is of the renewal type. The (external) means and squared coefficients of variation (scv) of the inter-arrival times are denoted by $1/\bar{\lambda}_{01}$, ca_{01} , $1/\bar{\lambda}_{06}$, ca_{06} respectively. The service times at each station/node as well as the inter-arrival times to each node cannot be approximated by exponential distributions. The jobs arriving to node i are processed according to the first come first served (FCFS) service protocol. There are 17 stations/nodes in the network where node i has s_i parallel servers. The service times at node i are independent and identically distributed with mean service time equal to $1/\bar{\mu}_i$ and scv equal to cs_i . The utilization of each node is less than or equal to unity. The problem is to determine the optimal configuration for the Geomatics division with the total number of servers S given.

3.1 Formulation

Among the earliest attempts to provide queueing network models are the exponential network results of Jackson [Jackson, 1957]. A Jackson network is a collection of queues with exponential service times in which the customers (jobs) travel from node to node according to transition probabilities given by a Markov chain. Customers (jobs/projects) from outside the network arrive according to a Poisson process and are served at each facility (with identical servers) subject to a first-come, first served discipline. In brief, if the network is stable and in steady state, a Jackson network can be treated as N independent multiserver birth and death queues where they are coupled via the traffic equations. The success of expressing the solution of Jackson network in product form has prompted researchers to approximate non-Markovian networks (i.e. where one cannot assure the conclusions of Poisson arrival streams and exponential service times) using a similar approach. The basic idea is to approximately characterize the arrival processes by two parameters and then to analyze the individual nodes separately. Many authors have developed two-parameter approximations for networks of queues, [Reiser and Kobayashi, 1974], [Kuehn, 1979], [Sevcik *et al*, 1977], [Chandy and Sauer, 1978], [Gelenbe and Mitrani, 1980] and [Shanthikumar and Buzacott, 1981]. Generally speaking, this method approximately characterizes each arrival and service process by two parameters: the arrival rate and its variability parameter; the mean service time and its variability parameter. The nodes are then analyzed as standard $GI/G/s$ queues partially characterized by the first two moments of the interarrival time and service time distributions. Our approach follows

the development of the Queueing Network Analyzer by [Whitt, 1983], whereby one can obtain the system performance measures by simply solving two systems of linear equations. Applying Whitt's Queueing Network Analyzer (QNA) concept, the first step is to solve for the flow rates, and the variability parameters at the internal arrival processes. Let

I_j = average arrival rate at node j ; I_{0j} = external arrival rate to node j ;
 λ_j = average service rate at node j ;
 ca_j = squared coefficient of variation (scv) for the arrival process at node j ; ca_{0j} is
the scv of the external arrival process to node j ;
 cs_j = squared coefficient of variation (scv) of the service time distribution at
node j ;
 R = routing matrix $[p_{ij}]$; p_{ij} is the probability that a job visiting station i
will visit station j after completion of service at i ;

$$I_i = I_{0i} + \sum_{j=1}^n I_j p_{ji} \quad i=1,2,\dots,n \quad (2)$$

$$I_i ca_i - \sum_{j=1}^n \{I_j (1 - r_j^2) p_{ji}^2 ca_j\} = I_{0i} ca_{0i} + \sum_{j=1}^n \{I_j p_{ji} (r_j^2 p_{ji} cs_j + 1 - p_{ji})\} \quad (3)$$

$i=1,2,\dots,n$

The system of equations (2) represents the traffic equations that estimate the composite mean arrival rates to each station. The second set of equations (3) provides the variability of the arrival process to each station, which is obtained by combining the three basic network operations: superposition (merging), thinning (splitting), and flow through a queue (departure). The derivation of the system of equations (3) is described in detail in [Whitt, 1983], [Bitran and Dasu, 1992] and once the I_i s are determined from equation (2), the system of equations (3) is linear in ca_i . Since the performance of the queue is estimated on the basis of the first two moments of the interarrival and service times, the necessary flow parameters have all been determined.

The performance measures at each station are next estimated using approximation formulae that are based on the first two moments of the interarrival and service times. A wide variety of approximations has been proposed for the analysis of $GI/G/s$ queues (see [Bitran and Dasu, 1992] for references therein). The remainder of this section will focus on the determination of the optimal configuration for the CF Geomatics organization using $GI/G/s$ approximations by [Whitt, 1983], [Allen, 1990]. Whitt's approximation formulae are based on the behaviour of $M/M/s$, $D/M/s$ and $M/D/s$ queues, heavy traffic approximations for $GI/G/s$ queues and computational experiments. The Allen-Cunneen approximation was developed by pattern recognition, not by formal proof. It is basically the Whitt formula but without the correction term and it is exact for the $M/M/s$, $M/G/1$ queueing systems. It gives reasonable good results for many queueing systems and is easier to compute. Table 2 summarizes Whitt, Allen-Cunneen approximation formulae for the $GI/G/s$ queues. In view of lemma 1, our objective will be to minimize the L norm on the waiting times in Figure 3.

Approximation Method	Waiting Time Formulation
Whitt	$W_q^{(GI/G/s)} = f \times \left(\frac{cs + ca}{2} \right) W_q^{(M/M/s)}$ $f = \begin{cases} \frac{4(ca - cs)}{4ca - 3cs} f_1 + \frac{cs}{4ca - 3cs} q & ; ca > cs \\ \frac{cs - ca}{2(ca + cs)} f_3 + \frac{cs + 3ca}{2(ca + cs)} q & ; ca \leq cs \end{cases}$ $q = \begin{cases} 1 & ; a > 1 \\ f_4^{2(1-a)} & ; 0 \leq a \leq 1 \end{cases} \quad \text{where } a = \frac{cs + ca}{2}$ $\begin{cases} f_1 = 1 + d \\ f_2 = 1 - 4d \\ f_3 = f_2 \times \exp\left(-\frac{2(1-r)}{3r}\right) \\ f_4 = \min\left\{1, \frac{f_1 + f_3}{2}\right\} \end{cases}$ $d = \min\left\{0.24, \frac{(1-r)(s-1)(\sqrt{4+5s}-2)}{16sr}\right\}$
Allen-Cunneen	$W_q^{(GI/G/s)} = \left(\frac{cs + ca}{2} \right) W_q^{(M/M/s)}$

Table 2 - Approximations for Waiting Time.

Lemma 2. The optimal configuration for the CF Geomatics organization is given by the solution of the following system, (S = the total number of servers),

$$\text{Min } L \text{ norm on } W_{q,i}^{(GI/G/s)} \text{ Min-Max } W_{q,i}^{(GI/G/s)}, \quad i=1,2,...,12,14,...,17 \quad (4)$$

s_i

subject to $r_i \leq 1$, $r_i = I_i/(s_i \lambda_i)$ (stability constraint) (5)

$$\sum_{j=1}^{17} s_j = S - s_{13}, \quad s_{13} = s_{12} \quad (\text{servers capacity constraint}) \quad (6)$$

$$j=1, j13$$

$$\mathbf{I}_i = \mathbf{I}_{0i} + \sum_{j=1, j13}^{17} \mathbf{I}_j p_{ji}, \quad i=1,2,\dots,n \quad (\text{traffic equations}) \quad (7)$$

$$\sum_{j=1, j13}^{17} \mathbf{I}_i c a_i - \sum \{ \mathbf{I}_j (1 - \mathbf{r}_j^2) p_{ji}^2 c a_j \} = \sum_{j=1, j13}^{17} \mathbf{I}_{0i} c a_{0i} + \sum_{i=1,2,\dots,12,14,\dots,17} \{ \mathbf{I}_j p_{ji} (\mathbf{r}_j^2 p_{ji} c s_j + 1 - p_{ji}) \} \quad (8)$$

(equations governing the variability of the arrival process)

s_j non-negative integers

and the expressions for $W_{q,i}^{(GI/G/s)}$ (the expected steady state time a job spends waiting at node i in the $GI/G/s$ queue) are defined in Table 2.

Since $W_{q,i}^{(GI/G/s)}$ is a nonlinear function, system (4 - 8) belongs to the class of nonlinear integer programming problems. The solutions of which are known to be notoriously difficult. Our solution approach is to replace $W_{q,i}^{(GI/G/s)}$ by the Allen-Cunneen or Whitt approximations and express the $W_{q,i}^{(M/M/s)}$ term using Erlang's busy function. The latter probability related function has been pre-tabulated in *LINGO6* software for any given s servers. Therefore $W_{q,i}^{(GI/G/s)}$ can be obtained as table-look-up values once s_i is specified.

For example $W_{q,i}^{(GI/G/s)} (c a_i + c s_i) W_{q,i}^{(M/M/s)}/2$ by Allen-Cunneen approximation
 $= (c a_i + c s_i) P(j - s_i) / (2(s_i \lambda_i - \tilde{e}_i))$

where $P(j - s_i)$ is the Erlang busy probability defined as

$$P(j - s_i) = (s \tilde{n})^s p_0 / s! (1 - \tilde{n}),$$

p_0 = steady state probability that no job is present in the system. Consequently, the system becomes a linear integer programming (IP) problem. (The same argument easily holds by using the Whitt approximation.)

A sequence of linear optimization problems is solved iteratively, with the initial guess of the solution given by the solution of Jackson network. Rewrite the objective function (4) by

$$\text{Min } \tilde{u} \quad \text{subject to } \tilde{u} \geq W_{q,i}^{(GI/G/s)} \text{ for all } i \quad (9)$$

The steps for the solution of system (9, 5, 6, 7, 8) are as follows.

1. Solve the traffic equations for \mathbf{I}_i .

$$\mathbf{I}_i = \mathbf{I}_{0i} + \sum_{j=1, j13}^{17} \mathbf{I}_j p_{ji} \quad i=1,2,\dots,12,14,\dots,17 \quad (7)$$

and denote the solution of the Jackson network as

v_i = optimal configuration (servers),

V_q = optimal value of the objective function.

2. $k = 0$; initialize $s_i(k)$ (server s_i in iteration k), $W_{q,i}^{(GLG/s)}(k)$ (queue time at station i in iteration k):

$$\begin{aligned} s_i(k) &= v_i ; \\ \dot{u}(k) &= V_q. \end{aligned}$$
3. $k = k + 1$; input $I_i, s_i(k-1)$ in

$$I_i ca_i - \sum_{j=1, j \neq i}^{17} \{ I_j (1 - r_j^2) p_{ji}^2 ca_j \} = I_{0i} ca_{0i} + \sum_{j=1, j \neq i}^{17} \{ I_j p_{ji} (r_j^2 p_{ji} cs_j + 1 - p_{ji}) \} \quad (3)$$

$$i=1,2,...,12,14,...,N$$

where $r_j = I_j / (s_j(k-1) \dot{u}_j)$ and solve for ca_i .
4. Solve the system (9,5,6,8) using *LINGO6*.
5. If $|\dot{u}(k) - \dot{u}(k-1)| / \dot{u}(k-1) \leq \text{tolerance factor}$, stop. Else, go to step 3.

Initially we are to consider the LP relaxation of system (6,5). (It is feasible to obtain the LP solution since Erlang's busy function tabulated in *LINGO6* has been extended to non-integer servers by linear interpretation.) The latter will provide insight on the feasibility of the IP solution space. Another interesting observation is that the LP relaxation allows us to examine the effect of having servers with ancillary activities [Hall, 1989], i.e. servers can alternate between different nodes/stations. The efficiency gains from this option should be obvious. What would otherwise be idle time becomes productive time. Additionally, it is well known that if the LP relaxation is infeasible, then so is the IP system (9,5,6,8). The feasible region to the LP relaxation problem is clearly convex and therefore a unique solution exists. Table 3 summarizes the difference between the solutions for the LP relaxation and the IP system (9,5,6,8) with the total number of servers 210, tolerance factor $1 \cdot 10^{-2} \cdot \dot{u}(k-1)$.

	Integer Solution	LP Relaxation Solution
Waiting Time (days) ($W_{q,1}, W_{q,2}, \dots, W_{q,17}$)	(0.0001, 0.0001, 0.0059, 0.0004, 0.0004, 1.641, 2.047, 3.612, 4.157, 3.909, 3.647, 3.554, 3.554, 3.606, 5.504, 4.100, 2.222)	(2.552, 2.552, 2.552, 2.552, 2.552, 1.302, 1.302, 2.474, 2.474, 2.604, 2.604, 2.604, 2.604, 2.604, 2.892, 2.748, 2.748)
Number of Servers ($s_1, s_2, s_3, \dots, s_{17}$)	(1, 1, 1, 1, 1, 10, 5, 77, 20, 5, 15, 24, 24, 11, 3, 6, 5)	(0.0134, 0.0133, 0.1006, 0.0258, 0.0244, 10.227, 5.274, 78.157, 20.735, 5.242, 15.415, 24.504, 24.504, 11.337, 3.274, 6.285, 4.866)

Table 3 – Comparison between LP Relaxation and IP Solution for 210 servers.

Sensitivity analysis for the LP relaxation problem can be easily conducted from the output of *LINGO6*. For example, the dual variable to the servers' constraint is 0.1515, which implies that the smallest upper bound on the waiting times for the network will be reduced by 0.1515 days for every unit increase in the total number of servers. On the other hand, parametric integer

programming is much more complicated than its LP counterpart. An alternative is to use approximate methods, such as the 'response surface methodology, where it involves fitting a hyperplane tangent to the response surface at a given arbitrary point [Ng and Lam, 1994]. Since the objective function (9) is a function of the servers and the scv of the service times, the response surface might be difficult to compute. However, the crux lies in whether it justifies devoting effort in using an approximate method on parametric programming to study queueing network models which employs approximations. In this paper, no sensitivity analysis will be performed on the IP system (9,5,6,7,8).

4. Conclusion

In this paper, we have proposed a mathematical model to evaluate the optimal system configuration to a hypothetical division in the Canadian Forces Geomatics organization devoted to the production of information in support of the U.S. National Imaging and Mapping Agency's (NIMA) Foundation Data Concept. In a continuation paper, given the optimal system configuration, the authors [Ng et al, 2002] have extended their model to examine the system behaviour of the Geomatics network. Bounding analysis in classical queueing theory is used extensively to acquire insights to the workflow analysis problem. In conclusion, the workforce configuration and workflow analysis provide some timely contributions towards the understanding of some of the challenges arising in military organizational studies. As a result, the analysis and solution of these studies are therefore of prime and utmost importance in resolving certain Command and Control issues.

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